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Some remarks on the risk

1. *The risk* belongs to the class of phenomena which are often described by physicists as the *emergent* ones. Such an emergent phenomenon has unique but measurable dimensions that are created by an extremely complicated character of an existing situation.

The definition of risk has never been explicitly worked out (*Yates, Stone* (1992)). The main reasons for this are - according to the authors - the lack of agreement about components creating risk as well as the impossibility of building a full *risk construct*. In addition, risk presents various characteristics in different situations which makes it even more difficult to diagnose, understand or define it fully, let alone the fact that according to people's subjective feelings the same situation may or may not create risk (*Williams, Nerendran* (1999)).

While analysing various definitions of risk, *March and Shapira* emphasised that risk is always connected with a decision-making problem which involves a question of optimising the expected profit. In general, all decision-making strategies base on a rule that a decision-making person prefers a higher expected profit at a stable level of risk or a lower level of risk at a constant profit value. Thus, if the expected value of profit increases, the connotations for a decision-making person are positive. If the level of risk increases, the connotations are negative.

The aim of this article is to present different decision-making strategies form the point of view of risk and possibilities to reach preferable purposes.

2. Let us start with the example that has been constructed for the needs of this article. Thanks to it, we would be able to carry out an introductory analysis of risk in a hypothetical but realisable situation.

In a company a salary for new workers is defined according to the following method. There are two urns: the first one contains two balls marked 900$ and 1100$, whereas in the second one there are two balls marked 300$ and 1900$. First, a new employee chooses one of the urns and afterwards draws a ball form the chosen urn. The inscription found on the ball will show the level of a future salary.

It seems that most people who would be asked to choose in this way would decide to draw a ball form the first urn. This strategy would be chosen because even if the drawing was

unfavourable (a salary of 900$) a worker would get an average salary, whereas the unfavourable drawing from the second urn (a salary of 300$) would make it difficult for the worker to make ends meet. This advantage is much more important than a chance of earning 1900$.

The above-described considerations are of psychological character but is it possible to confirm the reached conclusions with the use of more formal analysis?

Let us give the symbols X and Y to the level of salaries that can be drawn when we draw the balls from the first (X) and second (Y) urn. The salaries X and Y are the random variables. That is why, we are able to calculate their expected values, variances and standard deviations:

$EX = 1/2\ (900 + 1100) = 1000\$$, $EY = 1/2\ (300 + 1900) = 1100\$$,
$D^2X = 1/2\ [(900 - 1000)^2 + (1100 - 1000)^2\ ] = 10000\$^2$, $DX = 100\$$,
$D^2Y = 1/2\ [(300 - 1100)^2 + (1900 - 1100)^2\ ] = 64000\$^2$, $DY = 800\$$.

The calculated characteristics show that if we draw from the second urn, the expected value of salary is 10% higher than in a case of the first urn. The standard deviation is higher by 800%. Thus, if someone chooses to draw from the first urn, the formal explanation of this choice is the willingness to avoid the huge standard deviation, which here is the measurement of risk.

Paradoxically, if the first urn has been chosen, it is also favourable for the employer. If a company employs a lot of new workers and if they choose to draw form the first urn, it is able to save about 10% of its salary funds.

If it was possible to reach an agreement among workers, the most optimal strategy for them would be to choose the variant of the second urn each time and divide the sum of drawn salaries equally among everybody. Let's give the symbol Z_n to a level of salary possible to reach in this variant (where n is the number of workers who accepted this strategy). If the number of workers is quite high (let's say higher than 30), the normal distribution $N(1100\$, 800\sqrt{n}\ )$ will be a good approximation [1] of the salary distribution Z_n. The advantage of this process can be seen if we decide on e.g. $n = 50$ and look at the following probabilities:

$P\ \{Z_{50} > 900\} = 0.96$, $P\ \{Z_{50} > 1000\} = 0.81$, $P\ \{Z_{50} > 1100\} = 0.50$.

[1] Here we are using the *central limit theorem*.

3. A roulette player who bets their whole capital on the game behaves in an extremely risky way. Nevertheless, there are situations in which such behaviour is not groundless. Let's analyse the following example taken form the book written by *P. Billingsley* (1987).

The player enters a casino realising that till dawn he has to change his 100$ into 20000$. If he does not do it, he will be killed by the gangsters who he owns the money to. A successful roulette game is the only possibility to collect the necessary sum. So, the question whether to play or not basically does not exist. There is no other option. But how should the player play to maximise their chances of winning, that is to say, of surviving ?

It turns out that the optimal strategy in this situation will be the one of *the brave game* which bases on the following rule: the whole capital should be bet each time unless the won sum is higher than a target sum of 20000$. In our concrete situation the player's chance of leaving the casino with 20000$ is 0.0003 when they use the brave game strategy. If the player bet constant sums of money, particularly the small ones on the roulette game, their chance of surviving would decrease drastically.

4. People who gamble are described as the ones who take risk, but people who insure their lives, their houses or themselves against illnesses are thought to be careful. But if an insurance is treated as a *game* with an insurance company, it turns out that such a game, especially from the formal point of view is similar to the game between a gambler and a casino.

To exist and gain profits, a casino plays a *dishonest game* with its clients. The expected value of a player's winnings in the game is negative (e.g. in the roulette a player who bets 1$ will on average lose 1/19$). The situation between an insurance company and its customers can be described as a similar one. For obvious reasons, it plays a dishonest game with its clients (a sum of premiums is higher than the insurance paid by the firm). Thus, what is the difference between these institutions and their clients ?

- a person who insures their life etc. in an insurance company usually plays once a year, whereas money is bet many times in a casino. That is why, *the law of large numbers* [2] is of no importance for an insured person. In contrast, for a casino player who takes part in a low variance game and plays a lot of times the law of large numbers "makes" them lose;
- a casino player will receive a multiplied sum that they bet if they have *a good fortune*, an insurance company client – if they are *unlucky*. The winnings in both cases is of different psychological character.

[2] The law of large numbers has to be mentioned here. It says that the arithmetic mean of many observations which are characterised by the finite variance and identical probability distribution is approximately equal to the expected value of a single observation. The higher the number of observations, the better the estimation.

Let us analyse the importance of the above-mentioned law of large numbers for both institutions. It seems that the law has much greater meaning for casinos than for insurance firms. Both institutions take part in a large number of games which expected values are positive (from their point of view), but an insurance firm's games are, contrary to a casino's ones, independent. For instance, floods, fires, earthquakes etc. cause the necessity to pay insurance money to hundreds or thousands insured people. Therefore, *the issues of ruin* are more important than the law of large numbers for insurance companies. They allow to work out suitable strategies that will make it possible to survive despite of the fact that insurance firms suffer great periodic losses.

5. What conclusions can be drawn from the considerations mentioned in the article ? It seems that there are at least two theses:

- the decisions that seem to be risky are not always risky; it happens that they are the most rational ones,

- there are no universal methods which, regardless of the purposes we want to achieve will show the least risky strategies.

References:

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